

Confinement at Weak Coupling

Martin Schaden*

Rutgers University, 211 Smith Hall, 101 Warren St., Newark, NJ 07102, U.S.A.

The free energy of $U(N)$ and $SU(N)$ gauge theory was recently found to be of order N^0 to all orders of a perturbative expansion about a center-symmetric orbit of vanishing curvature. Here I consider extended models for which this expansion is perturbatively stable. The extreme case of an $SU(2)$ gauge theory whose configuration space is restricted to center-symmetric orbits has recently been investigated on the lattice [1]. In extension of my talk, a discussion and possible interpretation of the observed finite temperature phase transition is given. The transfer matrix of constrained $SU(N)$ lattice gauge theory is constructed for any finite temperature.

1. INTRODUCTION

The non-perturbative dynamics of $SU(N)$ gauge theories simplifies considerably for a large number of colors N [2] and confinement can be seen in a perturbative expansion of the free energy about a center-symmetric ground state configuration [3]. The center-symmetric perturbative vacuum has the same classical action as the usual perturbative vacuum for which the center symmetry is broken. However, the 1-loop free energy indicates that $SU(N)$ is in the broken phase [4] at weak coupling. The center symmetry thus is broken at sufficiently high temperatures and a perturbative center-symmetric expansion is somewhat formal. However, this (formal) perturbative expansion to all orders in the coupling defines a center-symmetric $1/N$ -expansion. I here consider some modified models whose center-symmetric phase is stable at any finite order of a weak coupling expansion. To mimic non-perturbative and higher order perturbative contributions to the free energy, external sources that couple to adjoint Polyakov loops are included [5]. Although such sources probably cannot be physically realized [6], the approach is analogous to studying a spin model in a magnetic field: the external sources favor and stabilize the weak coupling expansion about a particular (in this case center-symmetric) classical ground state.

The primary objective here is to construct

models that confine at weak coupling. Only minimal modifications to the standard Wilson action are considered that may achieve this. There in particular is no attempt to construct a general and realistic effective action for Polyakov loops [7].

In the limit of very large external sources, the configuration space consists of center-symmetric configurations only. Constraining the configuration space of $SU(2)$ -LGT to center-symmetric orbits only, was recently [1] found to nevertheless reproduce the full non-perturbative lattice β -function for $T \rightarrow 0$. Somewhat surprisingly, this constrained $SU(2)$ -model shows a violent increase in entropy at roughly the usual deconfinement temperature even though center-symmetry is enforced [1].

2. CENTER SYMMETRIC ORBITS

Center symmetric orbits are determined by their Polyakov loops. Consider a particular configuration of links $\{U\} = \{U_{x\mu} : U_{x\mu} \in SU(N), \forall x = (\mathbf{x}, \tau), \mu = 1, \dots, 4\}$ of a 4-dimensional periodic lattice with periods N_T and N_s on temporal and spatial cycles respectively. The temporal Polyakov loops, $\mathcal{P}^{(k)}(\mathbf{x})$, at a site $x = (\mathbf{x}, \tau)$ are,

$$\mathcal{P}^{(k)}(\mathbf{x}) = \text{Tr} U^k(x) \quad (1)$$

$$U(x) = U_{(\mathbf{x}, \tau)4} U_{(\mathbf{x}, \tau+1)4} \dots U_{(\mathbf{x}, \tau+N_T)4} \cdot (2)$$

*email: mschaden@andromeda.rutgers.edu

$\mathcal{P}^{(k)}(\mathbf{x})$ does not depend on the Euclidean time τ and

$$Q^{(k)}[\{U\}] := \sum_{\mathbf{x}} \mathcal{P}^{(k)}(\mathbf{x}) \mathcal{P}^{(k)\dagger}(\mathbf{x}) \geq 0, \quad (3)$$

may be thought of as gauge invariant topological charges of the configuration. The $Q^{(k)}$ are non-negative by definition and (up to a field-independent term) can be expressed by Polyakov loops in the adjoint representation. Of particular interest is $Q = Q^{(1)}$ given by ordinary Polyakov loops that wind just once around the temporal direction.

A (global) center transformation with the center element $z\mathbf{1} \in \mathbf{Z}_N \subset SU(N)$ maps $\{U\}$ to the configuration $\{U\}_z$ (up to possibly a periodic gauge transformation),

$$\{U\}_z = \begin{cases} U'_{(\mathbf{x},1)4} = zU_{(\mathbf{x},1)4}, \forall \mathbf{x} \text{ and} \\ U'_{x\mu} = U_{x\mu} \text{ otherwise} \end{cases} \quad (4)$$

The Wilson plaquette actions of $\{U\}$ and $\{U\}_z$ are identical. However, the center transformation of Eq.(4) *in general* is not equivalent to a gauge transformation of the configuration: gauge invariant Polyakov loops change by $\mathcal{P}^{(k)}(\mathbf{x}) \rightarrow z^k \mathcal{P}^{(k)}(\mathbf{x})$.

A configuration is center-symmetric, if any center transformation can be undone by a periodic gauge transformation. The gauge orbit represented by a center-symmetric configuration thus does not change under center transformations and we have that,

$$\text{center-symmetric } \{U\} \Leftrightarrow Q^{(1)}[\{U\}] = 0.$$

The definition of $Q = Q^{(1)}$ in Eq.(3) implies that *all* temporal Polyakov loops of a $Q = 0$ -configuration vanish. Since all contractible Wilson loops also are not affected by the center transformation, the orbit does not change. If on the other hand $Q > 0$, *some* Polyakov loop $\mathcal{P}^{(1)}(\mathbf{x}) \neq 0$. This gauge-invariant quantity acquires a phase and changes under center transformations. The orbit therefore changes when $Q[\{U\}] > 0$.

Note that the non-local nature of the charge Q in Eq.(3) is due to its gauge invariant definition only. With a gauge transformation temporal links on all but one particular temporal slice of a periodic lattice can be set to unity. In such an axial

gauge, Q is a quadratic function of the remaining non-trivial temporal links only. Due to the periodicity of the lattice, a gauge transformation that does not depend on the Euclidean time *diagonalizes* the remaining non-trivial temporal links and *orders* their eigenvalues. Finally, an Abelian gauge transformation will evenly distribute the diagonal temporal links in Euclidean time [3].

Center-symmetric gauge orbits of a lattice gauge theory (LGT) with $SU(N)$ structure group can thus be represented by standardized configurations with,

$$\begin{aligned} U_{x4} &= G = e^{2\pi i \bar{\theta}/N_T}, \text{ where} \\ \bar{\theta} &= \text{diag}\left(\frac{N-1}{2N}, \frac{N-3}{2N}, \dots, \frac{1-N}{2N}\right). \end{aligned} \quad (5)$$

The gauge transformation that brings a center-symmetric configuration to the standard form of Eq.(5) furthermore is unique up to Abelian gauge transformations that do not depend on Euclidean time. Since none of the eigenvalues of $U(x)$ are degenerate when $Q[\{U\}] = 0$, there is no monopole gauge fixing ambiguity [8] for center-symmetric orbits. They therefore do not describe monopole configurations. The results of [9] imply that the Pontryagin index of center-symmetric configurations vanishes as well.

Due to the complete determination of the temporal links in a particular axial (Polyakov) gauge, the transfer matrix method can be used to describe the dynamics of $SU(N)$ gauge theory constrained to center-symmetric orbits. There are two main differences to the transfer matrix for the space of gauge orbits in axial gauges of [10]. Because the constraint to center-symmetric orbits determines *all* temporal links of a periodic lattice uniquely in a particular gauge, one can formulate the partition function in terms of a transfer matrix even for periodic lattices of *finite* extent, i.e. when $T < \infty$. In addition, the residual gauge group is the group of time-independent Abelian gauge transformations only and Gauss's law for selecting physical states simplifies accordingly.

Identifying a configuration of spatial links $\{U_{(\mathbf{x},0)\mu}\}$ with a basis vector $|U\rangle$ of the Hilbert space, the transfer matrix elements of the constrained center-symmetric Yang-Mills theory are,

$$\langle U'|T|U\rangle = e^{\text{ReTr}\Phi(\{U',U\})}, \text{ with} \quad (6)$$

$$\Phi(\{U', U\}) = \beta_T \sum_{\mathbf{x}, i} (U'_{\mathbf{x}i}^\dagger G^\dagger U_{\mathbf{x}i} G) + \beta_s \sum_{j \neq i} P_{\mathbf{x}, ij}$$

where $P_{\mathbf{x}, ij}$ is the plaquette in the spatial ij -plane at \mathbf{x} . β_T and β_s are inverse coupling constants related to the temporal and spatial lattice spacing by dimensional transmutation. The dependence of the kinetic term on the diagonal matrix G given in Eq.(5) ensures that only center-symmetric configurations are generated by the transfer matrix of Eq.(6). Note that the parallel transport by G depends on N_T . For fixed lattice spacing, $G \sim \mathbf{1}$ at sufficiently low temperature and the transfer matrix in Eq.(6) essentially becomes the one of Creutz [10]. One is tempted to argue that the constraint to center-symmetric orbits is irrelevant at low temperatures and that the full $SU(N)$ model should therefore be recovered. But the residual gauge symmetry of the center-symmetric transfer matrix at any finite temperature $T > 0$ is just the Cartan subgroup rather than the full $SU(N)$ gauge group. The observation by [1] that a simulation of the constrained $SU(2)$ lattice model reproduces the Wilson loop expectation value of the full $SU(2)$ -LGT thus is rather encouraging.

Since all Polyakov loops with non-trivial N-ality vanish for a center-symmetric configuration, their expectation values cannot be used as order parameters of the constrained theory. The sudden increase in entropy of constrained $SU(2)$ found in [1] thus is not the manifestation of a phase with broken center symmetry, even though the transition occurs at roughly the deconfinement temperature of $SU(2)$ -LGT. The entropy density of the constrained model in fact exceeds the Stefan-Boltzmann limit for free massless gluons by an order of magnitude and is rather sensitive to the spatial lattice volume. This behavior of the truncated $SU(2)$ -model could be explained by a Hagedorn transition [11] at T_H . Instead of deconfining color charges, the string tension vanishes at T_H and a plethora of low-mass glueball states of arbitrary high spin is produced. The entropy density for $T > T_H$ in this case is limited by the infra-red cutoff of the finite lattice only.

3. PERTURBATIVE STABILITY

A Hagedorn transition at T_H can perhaps be described by the Nambu-Goto string model [12] and may even allow a perturbative expansion [13] of the constrained model just below T_H at large N . In support of weak coupling, the Stefan-Boltzmann bound apparently is attained by the constrained $SU(2)$ -model *just below or at* T_H (see Fig.3b of ref. [1]).

To better understand the behavior of models that confine at weak coupling, I will now consider the perturbative free energy of $U(N)$ and $SU(N)$ models with extended lattice actions,

$$S(\beta, \vec{\kappa}) = \beta S_W[\{U\}] + N_T \sum_{k=1}^{[N/2]} \kappa_k Q^{(k)}[\{U\}]. \quad (7)$$

Here $S_W[\{U\}]$ is Wilson's plaquette action. The charges $Q^{(k)}[\{U\}]$ are defined by Eq.(3) and the parameters $\kappa_k \geq 0, k = 1, \dots, [N/2]$ can be thought of as thermodynamic potentials or external sources².

To retain center symmetry, the Wilson action is extended by center-symmetric terms only. Note that Polyakov loops in the adjoint representation renormalize multiplicatively [2] and scale like the Wilson action at large N . Up to a field-independent normalization constant, the divergence may be absorbed in the parameters κ_k .

$\kappa_1 > 0$ emphasizes center-symmetric configurations and the extended models should therefore approach one with center-symmetric orbits only [1] for $\kappa_1 \sim \infty$. For $SU(N > 3)$ additional sources for charges $Q^{(2)} \dots Q^{([N/2])}$ are needed to have a well-defined weak coupling expansion in the center-symmetric phase. I include only the minimal number of external sources that stabilize the center-symmetric perturbative ground state.

The non-Wilson terms of the extended actions of Eq.(7) lift the degeneracy of the lattice ground state with respect to the center symmetry. $Q = Q^{(1)}[\{U\}] \geq 0$ breaks the degeneracy completely and for $\kappa_k > 0, k = 1, \dots, [N/2]$, configurations with minimal classical action are on center-symmetric orbits without curvature [3]. This center-symmetric orbit with lowest classical

² $[N/2]$ is the largest integer that satisfies $[N/2] \leq N/2$.

action furthermore is unique for a spatial lattice topology³ with contractible⁴ loops only.

$\kappa_1 > 0$ with $\kappa_i = 0$ for $i > 1$ is sufficient for a unique center-symmetric classical vacuum but does not stabilize the weak coupling expansion about this vacuum for $N > 3$. Although the minimum of $Q^{(1)}[\{U\}]$ occurs at the center-symmetric configurations, it is at least of third order in $N - 3$ directions. This can be seen by expanding $Q^{(k)}$ to second order in fluctuations about a center-symmetric configuration in the standard form of Eq.(5). Due to gauge invariance it is sufficient to consider Abelian fluctuations of the temporal links that do not depend on Euclidean time,

$$Q^{(k)}[\{\theta^i - \bar{\theta}^i = \delta^i\}] = \sum_{\substack{\text{spatial} \\ \text{sites } i}} \sum_{a,b=1}^N \delta_a^i H_{ab}^{(k)} \delta_b^i + \dots \quad (8)$$

with $N \times N$ Hessians,

$$H_{ab}^{(k)} = \cos(2\pi k(a - b)/N), \quad (9)$$

that are diagonal in the Fourier basis,

$$\theta_a^{(s)} = e^{2\pi i s a / N}, \quad j = 1, \dots, N. \quad (10)$$

$\theta^{(N)} = (1, \dots, 1)$ changes the determinant of the links and is a zero-mode of all $H^{(k)}$. $SU(N)$ fluctuations are in the $N - 1$ -dimensional sub-space spanned by $\{\theta^{(s)}; 0 < s < N\}$. For $N > 3$ the minimum of $Q = Q^{(1)}$ is of third and higher order in directions spanned by the eigenvectors $\theta^{(s)}$ with $s = 2, \dots, N - 2$. The Hessian $H^{(1)}$ thus has additional zero-modes when $N > 3$. To obtain a quadratic minimum suitable for a perturbative expansion about the center-symmetric configuration, the Wilson action should be extended by charges $Q^{(k)}$ with $k > 1$ for $SU(N > 3)$. For $SU(2)$ and $SU(3)$ lattice groups, an extension by $\kappa_1 Q^{(1)}[\{U\}]$ is sufficient.

³For instance the lattice equivalent of $S_3 \times S_1$. The spatial part of this lattice is the 3-dimensional surface of a 4-dimensional hyper-cube. Its 16 "corners" have (spatial) connection number five instead of six as for all other sites.

⁴On the lattice closed loops of links are considered similar when they differ in two consecutive links only. Two lattice loops are homotopic if a series of such similar loops interpolates between them. Finally, a lattice loop is contractible if it is homotopic to the "loop" without links. Plaquettes in this sense are contractible and Polyakov loops are not contractible on a periodic lattice.

4. THE 1-LOOP FREE ENERGY

Since it is gauge invariant, the 1-loop free energy is obtained [3,4] by expanding around a background of constant Abelian links,

$$\bar{U}_{x4} = \text{diag} (e^{2\pi i \theta_1 / N_T}, \dots, e^{2\pi i \theta_N / N_T}), \quad (11)$$

with $\bar{U}_{x1} = \bar{U}_{x2} = \bar{U}_{x3} = \mathbf{1}$. In the thermodynamic- and continuum- limit the free energy density of the extended $U(N)$ model at weak coupling is [3],

$$\begin{aligned} F_0(T, U(N)) = & 2(1 - N_A) \sum_{a,b=1}^N I(T, \theta_a - \theta_b; 0) \\ & - 4 \sum_{j=1}^{N_F} \sum_{a=1}^N I(T, \theta_a + 1/2; m_j) \quad (12) \\ & + \sum_{k=1}^{[N/2]} \tilde{\kappa}_k \sum_{a,b=1}^N \cos[2\pi k(\theta_a - \theta_b)]. \end{aligned}$$

N_A here is the number of Majorana fields in the adjoint representation satisfying *periodic* boundary conditions⁵ and N_F is the number of *anti-periodic* Dirac fields in the fundamental representation. The constants $\tilde{\kappa}_j$ have mass dimension four and are the (renormalized) continuum analogs of the κ_j in the lattice model. The function $I(T, \delta; m)$ is [3],

$$I(T, \delta; m) = -T^2 m^2 \sum_{n=1}^{\infty} \frac{\cos(2\pi n \delta)}{2\pi^2 n^2} K_2\left(\frac{nm}{T}\right), \quad (13)$$

where K_2 is a K-Bessel function normalized so that $K_2(|z| \sim 0) = 2/z^2$. In the limit of vanishing mass Eq.(13) therefore simplifies to,

$$I(T, \delta; 0) = -T^4 \sum_{n=1}^{\infty} \frac{\cos(2\pi n \delta)}{\pi^2 n^4}. \quad (14)$$

The one-loop free energy of the extended $SU(N)$ model is related to that of the extended $U(N)$ model by noting that the $U(1)$ -photon as well as one of the of the Majorana fields essentially decouple. To one loop one therefore has,

$$F_0(T, SU(N)) = F_0(T, U(N)) + \frac{\pi^2(1 - N_A)T^4}{45}. \quad (15)$$

⁵ $N_A = 1$ and $N_F = 0$ is ($N = 1$) supersymmetric Yang-Mills (SYM) and the Majorana particle in this case is the gaugino. Only $N_A = 0$ was considered in [3].

The correction term is of order N^0 and does not depend on the angles θ_a . However, the decoupled $U(1)$ -“photon” and Majorana fields contribute to the pressure and entropy density of the $U(N)$ -model only. This contribution in principle could be critical for *thermodynamic* stability which requires a positive entropy density and pressure. Since the decoupled fields give a phase-independent contribution to the free energy, one can ignore this physical requirement by assuming a contribution from “inert” degrees of freedom that are irrelevant for confinement but guarantee an overall positive pressure and entropy density⁶ In the following I therefore do not distinguish between the one-loop free energy of $U(N)$ - and $SU(N)$ - gauge models.

4.1. Perturbative Stability of the Free Energy

It is straightforward to verify that the center-symmetric configuration with $\theta_a = \bar{\theta}_a$ of Eq.(5) is an extremum of the 1-loop free energy of Eq.(12). It furthermore is well known [4] that this extremum is a *maximum* of the perturbative free energy for vanishing N_A and $\tilde{\kappa}_k$ ’s. The center-symmetric ground state in this case is not stable at weak coupling and the conclusion is that $SU(N)$ gauge theories are in a non-confining plasma phase at sufficiently high temperatures [4]. When $N_A > 1$ or the $\tilde{\kappa}_k$ are sufficiently large, this need not be the case.

Without Dirac fields, that is for $N_F = 0$, the center-symmetric configuration is *perturbatively* stable [14] when $N_A > 1$. Note that for supersymmetric Yang-Mills with $N_A = 1$, arbitrary small $\tilde{\kappa}_k > 0$ are capable of stabilizing the center-symmetric ground state at weak coupling. The exact perturbative cancellation between the gluon- and gaugino- contributions to the free energy is a manifestation of the supersymmetry of the model when $N_A = 1$. At finite temperature this cancellation occurs only for a gaugino satisfying *periodic* boundary conditions and the supersymmetry is (softly) broken by anti-periodic Majorana fields [15].

In the absence of massless *periodic* matter fields

⁶For $N_A = 0$ the “inert” degree of freedom could for instance simply be the physical photon.

in the adjoint representation, center-symmetric configurations minimize the 1-loop free energy of $SU(N)$ at (sufficiently) low temperatures when $\tilde{\kappa}_i > 0$. To obtain an upper bound for the critical temperature of these models, consider the Hessian of the 1-loop free energy at the center-symmetric configuration $\bar{\theta}_a$,

$$\begin{aligned} \mathcal{H}_{ab} &:= \frac{1}{8\pi^2 T^2} \frac{\partial^2 F_0}{\partial \theta_a \partial \theta_b} \Big|_{\theta=\bar{\theta}} \\ &= \frac{2T^2(1-N_A)}{\pi^2} \sum_{n=1}^{\infty} \left[\frac{\delta_{ab}}{Nn^2} - \frac{\cos\left(\frac{2\pi n(a-b)}{N}\right)}{n^2} \right] \\ &\quad + \sum_{n=1}^{[N/2]} \frac{n^2 \tilde{\kappa}_n}{T^2} \cos\left(\frac{2\pi n(a-b)}{N}\right) \\ &\quad - \sum_{j=1}^{N_F} \frac{m_j^2}{\pi^2} \sum_{n=1}^{\infty} \delta_{ab} \cos\left(\frac{(2a-1)\pi n}{N}\right) K_2\left(\frac{nm_j}{T}\right). \end{aligned} \quad (16)$$

Without matter in the fundamental representation ($N_F = 0$), \mathcal{H}_{ab} is diagonal in the Fourier basis of Eq.(10) with eigenvalues $h^{(j)}$ for $j = 1, \dots, N-1$ given by,

$$\begin{aligned} h^{(j)} &= \frac{T^2(1-N_A)}{6N} \left(1 - \frac{3}{\sin^2(j\pi/N)} \right) \\ &\quad + \frac{N}{2T^2} (j^2 \tilde{\kappa}_j + (N-j)^2 \tilde{\kappa}_{N-j}), \end{aligned} \quad (17)$$

where $\tilde{\kappa}_j = 0$ for $j > [N/2]$ and $j < 1$. A perturbative expansion about the center-symmetric orbit of minimal classical action requires that the Hessian is positive definite, or $h^{(j)} > 0, j = 1, \dots, N-1$. For $N_A > 1$ the center-symmetric model is stable with respect to small perturbations for vanishing $\vec{\kappa} = 0$ and at *all* temperatures $T > 0$. Any transition to a phase with broken center symmetry at $T_c < \infty$ in this case is of first order. For $N_A < 1$ on the other hand, the Hessian is positive for $T < T_p$ only, where

$$T_p^4 = \min_j \frac{N^2(j^2 \tilde{\kappa}_j + (N-j)^2 \tilde{\kappa}_{N-j})}{2(1-N_A)(\sin^{-2}(j\pi/N) - 1/3)}. \quad (18)$$

The $\tilde{\kappa}$ ’s thus limit the temperature range for which a weak coupling expansion about the center-symmetric background could be stable. For $SU(2)$ and $SU(3)$ gauge groups there is just

one $\tilde{\kappa}$ to contend with and Eq.(18) implies perturbative stability of the purely gluonic models with $N_A = N_F = 0$ for $T < T_p$ where

$$T_{p\,SU(2)}^4 = 6\tilde{\kappa}_1 ; \quad T_{p\,SU(3)}^4 = \frac{9}{2}\tilde{\kappa}_1 . \quad (19)$$

Note that since $\tilde{\kappa}_1 \rightarrow \infty$ essentially constrains the configuration space to center-symmetric orbits [1], the classical center-symmetric vacuum apparently is perturbatively stable at all temperatures in this limit. However, T_p is only an upper bound and a first order Hagedorn transition may, and apparently does [1], occur at a finite temperature $T_H < \infty = T_p$ in $SU(N)$ -LGT.

It perhaps is worth mentioning that T_p becomes independent of N if $\tilde{\kappa}_j(N) \propto (N \sin(j\pi/N))^{-2}$ for large N . The extended models thus are stable with respect to small perturbations over a finite range of temperature with $\tilde{\kappa}_j(N)$ of order N^0 .

4.2. The Phase Transition of the Extended U(2) and U(3) Models

Perturbative stability is necessary, but not sufficient for thermodynamic stability. However, for $\tilde{\kappa}_j > 0$, the global minimum of the free energy of the extended $SU(N)$ models at sufficiently small coupling is center symmetric for $T \sim 0$. If this perturbatively stable state turns into a *local* minimum at a temperature $T_c < T_p$, a first order transition to the true minimum of the free energy may occur. At sufficiently weak coupling, the center symmetry then would be broken for temperatures above T_c .

The free energy of the extended $U(2)$ model with no quark flavors depends on just one angle $\theta = \theta_2 - \theta_1$. The 1-loop free energy is,

$$F_0(T, U(2)) = -8(1 - N_A)T^4 \sum_{n=1}^{\infty} \frac{\cos^2(\pi n \theta)}{\pi^2 n^4} + 4\tilde{\kappa}_1 \cos^2(\pi \theta) . \quad (20)$$

Its extrema are at half-integer values of θ and the center-symmetric value of $\theta = 1/2$ is the absolute minimum at weak coupling for $T < T_c$, where

$$T_c^4{}_{SU(2)} = \frac{48}{\pi^2}\tilde{\kappa}_1 < T_p^4{}_{SU(2)} = 6\tilde{\kappa}_1 . \quad (21)$$

The first order transition of the extended $SU(2)$ model occurs just below the temperature T_p at

which the center-symmetric state anyway becomes unstable with respect to small fluctuations. Assuming that the scale $\tilde{\kappa}_1$ does not change drastically, the two temperatures are reasonably close, $T_c = (8/\pi^2) \sim 0.95 T_p$.

Note that universality arguments and lattice simulations suggest that the deconfinement transition of $SU(2)$ -LGT is of second order [16]. However, these arguments presume the existence of a mass gap which does not arise at finite orders of perturbation theory.

The analysis is more involved for $SU(N > 2)$ and I only give results for the extended $U(3)$ model. The free energy of $U(3)$ depends on two independent angles, $\theta = \theta_1 - \theta_2 \leq \phi = \theta_1 - \theta_3$. For $N_F = 0$ and at 1-loop it is of the form,

$$F_0(T, U(3)) = \tilde{\kappa}_1 \left| 1 + e^{2\pi i \theta} + e^{2\pi i \phi} \right|^2 - 2T^4(1 - N_A) \sum_{n=1}^{\infty} \frac{\left| 1 + e^{2\pi i n \theta} + e^{2\pi i n \phi} \right|^2}{\pi^2 n^4} . \quad (22)$$

At weak coupling the center symmetry is broken in a *first* order transition at $T_{c_1\,SU(3)} \sim 0.92 T_{p\,SU(3)}$. This transition is rather similar to that of the $SU(2)$ -model and in fact is to a $U(2)$ -invariant vacuum configuration. After the transition, the Polyakov loop reaches 1/3 of its maximal value only. A subsequent first order transition to the perturbative $SU(3)$ -invariant vacuum occurs at $T_{c_2\,SU(3)} \sim 1.02 T_{p\,SU(3)}$. Again assuming that $\kappa_1(SU(3))$ does not change appreciably, the two transitions are only about 10% apart, that is $T_{c_2\,SU(3)} \sim 1.10 T_{c_1\,SU(3)}$.

5. DISCUSSION

At low temperatures all the $U(N)$ and $SU(N)$ models considered here have a stable *perturbative* ground state that is invariant under the global $Z(N)$ center symmetry. The expectation value of the Polyakov-loop vanishes to all orders in perturbation theory in this low-temperature phase which "confines" static color charges in this sense. However, there is no mass gap at any finite order in the weak coupling expansion and all excitations are massless. The effective number of thermodynamic degrees of freedom in the perturbative center-symmetric phase nevertheless almost van-

ishes: Eq.(20) implies less than 0.25 thermodynamic degrees of freedom for $U(2)$ in the perturbative center-symmetric phase – and $U(N > 2)$ has even fewer [3]).

In these models confinement due to center-symmetry apparently is quite separate from the existence of a mass gap. The latter might arise due to a distinct (non-perturbative) mechanism. At weak coupling, the correlations due to the non-trivial ground state necessarily are long range and universality arguments that relate the behavior near phase-transitions to spin models with short-range interactions fail. Indeed, the phase transition at *weak coupling* is expected to be of first order [16] also in $SU(2)$ -LGT. The extended $SU(3)$ -model at weak coupling shows two separate first order phase transitions at temperatures $T_{c2} \sim 1.1 T_c$. In the intermediate phase the Polyakov loop attains only 1/3 of its maximal possible value.

The free energy to one loop was computed in the presence of certain external sources. Insofar as these sources mimic higher order perturbative and non-perturbative corrections, the observed pattern of phase transitions perhaps is a qualitative possibility. It thus is noteworthy that the Polyakov loop jumps to just $40 \pm 10\%$ of its maximal possible value at the deconfinement transition in lattice simulations [17] and increases rather slowly with the temperature thereafter. Heavy ion experiments also do not observe a gas of free quarks and gluons near the phase transition [18].

That severe infrared divergences [19] may prohibit a fully broken perturbative ground state at high temperatures could qualitatively explain some of these observations. The infrared divergence would eliminate the option of a first order transition to the fully broken perturbative vacuum. One therefore might expect that the corresponding phase transition moves to higher temperatures and eventually disappears altogether in a calculation of the free energy to higher *perturbative* order. In $SU(2)$ the remaining possibility at weak coupling would be a second order transition. In $U(3)$ (and presumably also $U(N > 2)$) a first order transition to a state with broken center-symmetry (but non-maximal expectation of the

Polyakov loop) would remain a possibility. The pattern of symmetry breaking of the extended models therefore thus could be more realistic if a transition to the trivial vacuum can be excluded at weak coupling. Phenomenologically, this (perturbative) restriction on the allowed phase transitions can be simulated by a Sutherland potential for the eigenvalues [20] or a (weaker) logarithmic repulsion [21].

Recently an extreme version of the extended models was studied on the lattice [1]. The configuration space of an $SU(2)$ -LGT in this case was constrained to center-symmetric orbits only. This constrained $SU(2)$ -LGT reproduces the non-perturbative β -function of full $SU(2)$ -LGT at $T \sim 0$ but instead of a deconfinement phase transition appears to have a Hagedorn-transition [11]. The constrained model may therefore be more amenable to a string description [12] than usual LGT. On a periodic lattice the constraint on the configuration space can be readily implemented in a particular (ghost-free) Polyakov gauge for any $SU(N)$ -LGT. The transfer-matrix of the model (Eq.(6)) can be formulated at any finite temperature. A Hamiltonian construction of constrained models at large N could be of considerable interest, since they confine at weak coupling [3,13].

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